



Selected Topics in Cryptography

Guía de estudio para el ETS

1. Find a non-singular elliptic curve over the \mathbb{Z}_5 . Justify why is it non-singular. **(2 points)**
2. Let E be an elliptic curve defined over \mathbb{Z}_5 : $y^2 = x^3 + 3x + 3$
 - a) Compute all points on E over \mathbb{Z}_5 .
 - b) How many points does E have?
 - c) Find a generator point for this elliptic curve. Justify why it is a generator point.
3. Given the elliptic curve $y^2 = x^3 + x + 3 \pmod{7}$ with the points, compute the following operations
 - a) Verify that $P = (4, 1) \in E(1, 3)$. If your answer is positive, compute $-P$ and $2P$
 - b) if $P = (6, 6)$ calculate $P + \mathcal{O}$
 - c) $(6, 1) + (6, 6)$
 - d) $(5, 0) + (4, 1)$
4. Compute a session key between two entities Alice and Bob, in a ECDH protocol. Your secret vale is $a = 5$. You receive from Bob $B = (3, 2)$. The elliptic curve being used is defined by $y^2 = x^3 + x + 4 \pmod{5}$.

$$\begin{aligned} G &= (2, 2) \\ 2G &= (0, 2) \\ 3G &= (3, 3) \\ 4G &= (1, 4) \\ 5G &= (1, 1) \\ 6G &= (3, 2) \\ 7G &= (0, 3) \\ 8G &= (2, 3) \\ 9G &= \mathcal{O} \end{aligned}$$

5. Consider the public key $K_{pb} = (p, a, b, q, A, B) = (7, 1, 1, 5, (2, 5), (0, 6))$ for ECDSA:, if $h(x) = 4$, verify the signature $(r, s) = (0, 3)$
6. Consider ECDSA, show why the signature (r, s) satisfies the condition $r = x_p \pmod{q}$ where x_p is the x coordinate of $P = u_1A + u_2B$, A is generator and B is the public key.
7. An RSA encryption scheme has the set-up parameters $p = 17$ and $q = 19$. The public key is $e = 5$
 - a) Decrypt the ciphertext $y = 2$ using Chinese Remainder Theorem (CRT).
 - b) Verify your result by encrypting the plaintext without using the CRT.

8. What is the purpose of RSA-PSS? Please list the differences between schoolbook RSA and RSA-PSS?
9. Considering EdDSA answer the following questions:
 - a) Which cryptographic service provide EdDSA?
 - b) Why is this cryptographic algorithm secure? Please describe the intractable mathematical problem that provide security to EdDSA.
 - c) List the differences between the parameters (elliptic curve, finite field, etc) used for ECDSA and the parameters used for EdDSA.
10. The protocol TLS 1.3 provides security over a computer network. To do this it uses several cryptographic algorithms.
 - a) List the cryptographic algorithms used in this protocol to provide integrity.
 - b) Why TLS 1.3 provide perfect forward secrecy (PFS)? Which cryptographic mechanism provide PFS and why?
 - c) Which cryptographic algorithms are used for key exchange in TLS 1.3?
 - d) What would happen if we do not use digital certificates in TLS?
 - e) What is the purpose of a certificate authority?